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SPATIALLY CONTINUOUS MIXED P_2-P_1 SOLUTIONS FOR PLANAR GEOMETRY

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INTRODUCTION

Even-order Legendre polynomial (P_N) expansion approximations of the neutron transport equation have historically seen only limited practical application. Research in the last decade [1] has resolved one of the historical theoretical objections [2] to the use of even-order P_N approximations in planar geometry, namely the ambiguity in the prescription of boundary conditions as a result of an odd number of unknowns. This research also demonstrated the P_2 approximation to be more accurate than the P_1 approximation in planar geometry away from boundary layers and material interfaces. Neither the P_1 nor the P_2 approximation is convincingly more accurate near material interfaces. This progress motivated the reexamination of the multidimensional simplified P_2 (SP_2) approximation [3], the development of P_2 approximations for planar geometry stochastic transport problems [4], and the examination of the P_2 and SP_2 approximations as a synthetic acceleration technique for the discrete ordinates equations. [5]

The major remaining objection to even-order P_N approximations is that the scalar flux distributions obtained using these approximations can exhibit large spatial discontinuities at material interfaces and source discontinuities. In contrast, the odd-order P_N approximations typically utilized give spatially continuous scalar flux distributions at these locations. In this paper, we propose a mixed P_2-P_1 angular approximation designed to take advantage of the improved accuracy of the P_2 approximation in the interior of material regions and near external boundaries while retaining the continuous solutions obtained by the P_1 approximation near material interfaces and source discontinuities. We present numerical results from a series of eigenvalue calculations to demonstrate the accuracy of the mixed P_2-P_1 angular approximation.

THE MIXED P_2-P_1 ANGULAR APPROXIMATION

The P_2 angular approximation to the planar geometry neutron transport equation in a spatial domain $0 \le x \le L$ can be written as [1]:

$$-\frac{d}{dx}\frac{1}{3\sigma_{a1}(x)}\frac{d}{dx}\hat{\phi}(x)+\hat{\sigma}_{a0}(x)\hat{\phi}(x)=\hat{Q}(x) \quad , \qquad (1)$$

where

$$\hat{\sigma}_{a0}(x)=\frac{\sigma_{a0}(x)}{1+\frac{4}{5}\rho(x)} \quad , \qquad (2)$$

$$\hat{Q}(x)=\frac{Q(x)}{1+\frac{4}{5}\rho(x)} \quad , \qquad (3)$$

$$\hat{\phi}(x)=\left[1+\frac{4}{5}\rho(x)\right]\phi_0(x)-\frac{4}{5}\rho(x)\frac{Q(x)}{\sigma_{a0}(x)} \quad , \qquad (4)$$

and

$$\rho(x)=\frac{\sigma_{a0}(x)}{\sigma_{a2}(x)} \quad . \qquad (5)$$

Here $Q(x)$ is a neutron source, $\sigma_{an}(x)=\sigma_t(x)-\sigma_{sn}(x)$, $n=0,1,2$, are the "absorption" cross sections obtained by subtracting the Legendre angular moments of the differential scattering cross section from the total cross section, and $\phi_0(x)$ is the neutron scalar flux. Eq. (1) is a standard diffusion equation (with a modified absorption cross section and source) for the modified scalar flux unknown $\hat{\phi}(x)$. The neutron scalar flux, $\phi_0(x)$, is readily obtained from the unknown $\hat{\phi}(x)$ using Eq. (4). Defining the parameter $\rho(x)$ by Eq. (5) gives the P_2 approximation and setting $\rho(x)=0$ results in the standard P_1 approximation. Marshak vacuum boundary conditions for the P_2 approximation are given by [1]

$$\frac{1}{4}\left(\frac{1+\frac{1}{2}\rho(x_b)}{1+\frac{4}{5}\rho(x_b)}\right)\hat{\phi}(x_b)\mp\frac{1}{6\sigma_{a1}(x_b)}\frac{d}{dx}\hat{\phi}(x_b)= -\frac{3}{40}\frac{\rho(x_b)}{\sigma_{a0}(x_b)}\hat{Q}(x_b) \quad , \quad \begin{cases} x_b=0 \\ x_b=L \end{cases} \quad . \qquad (6)$$

In this paper, we propose to define the parameter $\rho(x)$ by the equation

$$\rho(x)=\frac{\sigma_{a0}(x)}{\sigma_{a2}(x)}\left[1-\exp(-p\,\sigma_t(x)d_i(x))\right] \quad , \qquad (7)$$

where p is a user-specified parameter and $d_i(x)$ is the distance to the nearest material interface or source discontinuity at a given spatial point x. For $p=0$, the pure P_1 approximation is recovered for all x; for $p\to\infty$, the pure P_2 approximation is obtained for all x. With this representation for $\rho(x)$ and $1 \le p << \infty$ (e.g. $p \approx 5$), the P_1 approximation is used at material interfaces and source discontinuities and the angular approximation transitions smoothly to the P_2 approximation within approximately one mean free path from the material interface or source discontinuity. The numerical solution of the mixed P_2-P_1 equations requires essentially the same computational expense as the solution of the standard P_1 equations.

NUMERICAL RESULTS

In this section, we compare the accuracy of the mixed P_2-P_1 angular approximation to the pure P_1 and P_2 angular approximations for a series of two-region k-eigenvalue problems (numerical test problem IV.F of Ref. [1]). The test problem is a two-region, isotropically scattering critical slab 5.0 cm thick, with vacuum left and right boundaries and cross-section discontinuities at x = 2.5 cm. The mathematical description of the transport problem is

$$\mu \frac{\partial}{\partial x}\psi(x,\mu)+\sigma_t\psi(x,\mu)= \frac{1}{2}\left[\sigma_s(x)+\frac{\nu\sigma_f(x)}{k}\right]\int_{-1}^{1}\psi(x,\mu')d\mu' \quad , \quad 0<x<5 \quad , \tag{8a}$$

$$\psi(0,\mu)=0 \quad , \quad 0<\mu\le 1 \quad , \tag{8b}$$

$$\psi(5,\mu)=0 \quad , \quad -1\le\mu<0 \quad . \tag{8c}$$

The material properties are σ_t = 1.0 cm^{-1} across the two regions of the slab, $\sigma_s(x)=\sigma_s^l = \textit{variable}$ in the left region, $0 \le x < 2.5$ (see Table I), and $\sigma_s(x)=\sigma_s^r =$ 0.99 cm^{-1} in the right region, $2.5 < x \le 5$. The fission cross section $\nu\sigma_f(x)$ takes the values $\nu\sigma_f^l$ = 0 in the left region and $\nu\sigma_f^r = \nu\sigma_f^{critical}$ in the right region of the slab. The critical values $\nu\sigma_f^{critical}$ for each value of σ_s^l are given in Table I (taken from Ref. [1]) and give S_{16} discrete ordinates transport values of $k=1$.

The percent relative errors in the computed k eigenvalue obtained using the pure P_1 and P_2 angular approximations to Eqs. (8) are shown in Table I. The accuracy of both approximations degrades as σ_s^l decreases, i.e. as the left region becomes less diffusive and the magnitude of the material property discontinuity increases. However, the eigenvalues obtained using the P_2 approximation are significantly more accurate than those obtained using the P_1 approximation for all values of σ_s^l. The scalar flux distributions surrounding the material discontinuity are shown in Fig. 1. The P_1 scalar flux is continuous at the material interface but is significantly in error both near the interface and away from the interface. The P_2 scalar flux is discontinuous at the material interface but is more accurate than the P_1 solution away from the interface.

The transport problem was also solved using the mixed P_2-P_1 approximation with three values of the parameter p of Eq. (7), p = 2.5, 5, and 10. The scalar flux distributions near the material interface obtained using the mixed P_2-P_1 approximation are shown in Fig. 1. Larger values of p give scalar flux distributions with much of the character of the P_2 solution except they are spatially continuous at the material interface. Smaller values of p give more diffusive scalar flux solutions. The percent relative errors for the k eigenvalues obtained using the mixed P_2-P_1 approximation are shown in Table I. The mixed P_2-P_1 approximation is more accurate than the P_1 approximation for all values of the parameter p and σ_s^l considered. For this test problem, smaller values of the parameter p give more accurate results for the k eigenvalue than larger values. With the exception of the $\sigma_s^l = 0.99$ case, the mixed P_2-P_1 approximation is more accurate than both the P_1 and the P_2 approximations for all values of the parameter p.

CONCLUSIONS

In this paper, we showed that the P_2 approximation could be modified to give spatially continuous scalar flux solutions at material interfaces and source discontinuities. The proposed modification uses the P_1 angular approximation at material interfaces and source discontinuities and smoothly transitions to the P_2 approximation within approximately a mean free path from the interface. Neither the P_1 nor the P_2 approximation is convincingly more accurate near material interfaces, so the P_1 approximation can be utilized at these locations to give spatially continuous results. Numerical results from a series of k-eigenvalue test problems demonstrate that the mixed P_2-P_1 approximation can yield improvements in accuracy over both the pure P_1 and P_2 approximations employed alone.

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TABLE I. Percent Relative Errors in the Criticality Eigenvalue k.

σ_s^l	$\nu\sigma_f^r = \nu\sigma_f^{critical}$	P_1	P_2	mixed P_2-P_1 $p = 2.5$	$p = 5$	$p = 10$
0.99	0.1408299	-8.54	+0.19	-1.66	-0.70	-0.23
0.90	0.1650953	-9.86	+1.58	-0.68	+0.50	+1.07
0.80	0.1803974	-11.13	+2.57	-0.20	+1.21	+1.90
0.70	0.1903208	-12.09	+3.29	+0.06	+1.65	+2.46
0.60	0.1974249	-12.84	+3.86	+0.21	+1.95	+2.87
0.50	0.2028380	-13.45	+4.34	+0.30	+2.16	+3.19
0.40	0.2071421	-13.96	+4.76	+0.35	+2.33	+3.44
0.30	0.2106719	-14.40	+5.12	+0.38	+2.46	+3.66
0.20	0.2136351	-14.79	+5.45	+0.39	+2.56	+3.84
0.15	0.2149489	-14.96	+5.60	+0.40	+2.61	+3.92

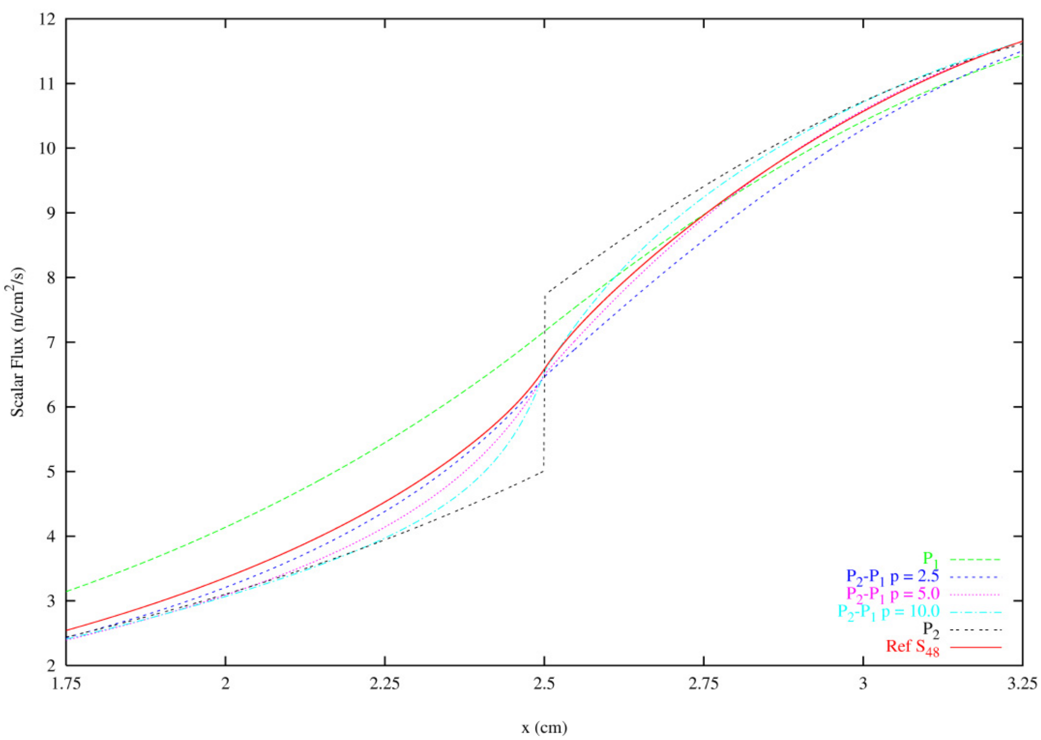

Fig. 1. Scalar flux near material property discontinuity for $\sigma_s^l = 0.6$ case.

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